# MAGNETOACOUSTIC WAVES IN TWO-COMPONENT MEDIA

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We consider plane waves in a conducting two-component medium in the presence of an external magnetic field. The medium consists of an elastic and a fluid component (porous medium with an elastic skeleton, where the pores are filled with a viscous compressible fluid). Pore sizes and the dimensions of the solid particles are assumed small compared with the distance over which the kinematic and dynamic characteristics of the motion undergo a significant change, and both components of the medium can, therefore, be assumed continuous. The dynamics of such a medium (in the absence of a magnetic field) was studied in [1 – 3]. In [4] it was shown, that the Biot equations [2] have the widest applicability in the case of harmonic waves.

Below we use the Biot models and the equations of electromagnetic field, to study the wave motion of the given medium in the presence of a constant, homogeneous magnetic field **H**. We note the existence of six types of waves, four of them polarized in the plane containing the wave vector **k** and the vector **H**, and the remaining two polarized in the direction perpendicular to this plane. We obtain expressions for their phase velocities and the coefficients of absorption in the critical case of a weak magnetic field.

1. Linearized equations of motion of the two-component conducting media, have the form

$$\rho_{11}\frac{\partial \mathbf{v}_1}{\partial t}+\rho_{12}\frac{\partial \mathbf{v}_2}{\partial t}+\beta(\mathbf{v}_1-\mathbf{v}_2)=(\lambda+\mu)\,\mathrm{grad\,div}\,\mathbf{U}_1+ +\mu\Delta\mathbf{U}_1+Q\,\mathrm{grad\,div}\,\mathbf{U}_2-\frac{1}{4\pi}\mathbf{H}\times\mathrm{rot}\,\mathbf{h}_1 \tag{1.1}$$

$$\rho_{12}\frac{\partial \mathbf{v}_1}{\partial t}+\rho_{22}\frac{\partial \mathbf{v}_2}{\partial t}-\beta(\mathbf{v}_1-\mathbf{v}_2)=Q\,\mathrm{grad\,div}\,\mathbf{U}_1+R\,\mathrm{grad\,div}\,\mathbf{U}_2-\frac{1}{4\pi}\mathbf{H}\times\mathrm{rot}\,\mathbf{h}_2$$

$$\rho_{11}=\rho_1-\rho_{12},\qquad \rho_{22}=\rho_2-\rho_{12},\qquad \beta=\mu_f m^2/\varkappa$$

Here $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{U}_1$, $\mathbf{U}_2$ are the velocity and displacement vectors of the elastic and fluid component, respectively, ($\mathbf{U}_\nu=(i/\omega)\,\mathbf{v}_\nu$, $\nu=1,2$); $\rho_1$ and $\rho_2$ are their respective masses per unit volume of the medium, $\rho_{12}$ is the dynamic coupling coefficient ($\rho_{12}<0$); $\lambda$, $\mu$, $Q$ and $R$ are elastic constants, $\mathbf{h}_1$ and $\mathbf{h}_2$ denote small variations of the magnetic field intensity within the wave in the respective components of the medium, $\mu_f$ is the viscosity of the fluid, $m$ and $\varkappa$ denote the porosity and permeability of the elastic skeleton. When $\mathbf{H}=0$, Eqs (1.1) coincide with the Biot equations.

Current appearing in each of the components during the motion of the medium across the magnetic field, consists of the induced current and the leakage current from the other component. Thus we have

$$\mathbf{j}_1=\eta_1(\mathbf{E}_1+(1/c)\mathbf{v}_1\times\mathbf{H})+\mathbf{j}_{12},\qquad \mathbf{j}_2=\eta_2(\mathbf{E}_2+(1/c)\mathbf{v}_1\times\mathbf{H})-\mathbf{j}_{12} \tag{1.2}$$

where $\mathbf{j}_1$ and $\mathbf{j}_2$ denote the currents in the first and second component per unit cross section of the medium, $\eta_1$ and $\eta_2$ are the coefficients of electrical conductivity, $\mathbf{E}_1$ and $\mathbf{E}_2$ denote the respective induced electric field intensities, $c$ is the velocity of light and $\mathbf{j}_{12}$ denotes the current leaking from the second component into the first.

Amount of energy appearing as Joule heat in unit time, is

$$W = \frac{j_1^2}{\eta_1} + \frac{j_2^2}{\eta_2}$$

and the condition that this energy is at minimum for fixed $E_\nu$ and $\mathbf{v}_\nu$, yield

$$\mathbf{j}_{12} = \frac{\eta_1\eta_2}{\eta_1 + \eta_2}\left[\mathbf{E}_2 - \mathbf{E}_1 + \frac{1}{c}(\mathbf{v}_2 - \mathbf{v}_1) \times \mathbf{H}\right] \qquad (1.3)$$

Relations (1.2) then become

$$\mathbf{j}_\nu = \eta_\nu \mathbf{E}, \qquad \mathbf{E} = \frac{1}{\eta_1 + \eta_2}\left[\eta_1\left(\mathbf{E}_1 + \frac{1}{c}\mathbf{v}_1 \times \mathbf{H}\right) + \eta_2\left(\mathbf{E}_2 + \frac{1}{c}\mathbf{v}_2 \times \mathbf{H}\right)\right] \qquad (1.4)$$

and the vector $E$ can be regarded as the effective electric field strength in a two-component medium.

The Maxwell's equations

$$\operatorname{rot} \mathbf{E}_\nu = -\frac{1}{c}\frac{\partial \mathbf{h}_\nu}{\partial t}, \qquad \operatorname{rot} \mathbf{h}_\nu = \frac{4\pi}{c}\mathbf{j}_\nu, \qquad \operatorname{div} \mathbf{h}_\nu = 0$$

with (1.4) taken into account yield

$$\frac{\partial \mathbf{h}_1}{\partial t} = \frac{\eta_1}{\eta_1^2 + \eta_2^2}\left[\eta_1 \operatorname{rot}(\mathbf{v}_1 \times \mathbf{H}) + \eta_2 \operatorname{rot}(\mathbf{v}_2 \times \mathbf{H}) + \frac{c^2}{4\pi}\left(1 + \frac{\eta_2}{\eta_1}\right)\Delta \mathbf{h}_1\right], \qquad \eta_2\mathbf{h}_1 = \eta_1\mathbf{h}_2 \qquad (1.5)$$

Eqs. (1.1) and (1.5) together form a closed system. We shall seek its solution in the form $\exp\{i[(\mathbf{kr}) - \omega t]\}$ describing the propagation of plane waves, with the wave vector $\mathbf{k}$ and the frequency $\omega$. In addition, (1.1) and (1.5) can be reduced to the following system of algebraic equations:

$$\omega^2\left[\left(\rho_{11} + i\frac{\beta}{\omega}\right)\mathbf{v}_1 + \left(\rho_{12} - i\frac{\beta}{\omega}\right)\mathbf{v}_2\right] = (\lambda + \mu)\mathbf{k}(\mathbf{kv}_1) + \mu k^2\mathbf{v}_1 + Q\mathbf{k}(\mathbf{kv}_2) + (\omega/4\pi)\mathbf{H} \times (\mathbf{k} \times \mathbf{h}_1) \qquad (1.6)$$

$$\omega^2\left[\left(\rho_{12} - i\frac{\beta}{\omega}\right)\mathbf{v}_1 + \left(\rho_{22} + i\frac{\beta}{\omega}\right)\mathbf{v}_2\right] = Q\mathbf{k}(\mathbf{kv}_1) + R\mathbf{k}(\mathbf{kv}_2) + \frac{\omega}{4\pi}\mathbf{H} \times (\mathbf{k} \times \mathbf{h}_2)$$

$$\left[\omega + i\frac{k^2c^2(\eta_1 + \eta_2)}{4\pi(\eta_1^2 + \eta_2^2)}\right]\mathbf{h}_1 + \frac{\eta_1}{\eta_1^2 + \eta_2^2}[\eta_1\mathbf{k} \times (\mathbf{v}_1 \times \mathbf{H}) + \eta_2\mathbf{k} \times (\mathbf{v}_2 \times \mathbf{H})] = 0$$

Let us choose the coordinate axes in such a manner, that the $x$-axis is directed along the wave vector $\mathbf{k}$, while the vector $\mathbf{H}$ lies in the $xy$-plane. Then the projections of (1.6) on these axes are

$$[(\gamma_{11} + i\gamma)u - \sigma_{11}]v_{1x} + [(\gamma_{12} - i\gamma)u - \sigma_{12}]v_{2x} - \alpha H_y\sqrt{u}h_{1y} = 0$$

$$[(\gamma_{12} - i\gamma)u - \sigma_{12}]v_{1x} + [(\gamma_{22} + i\gamma)u - \sigma_{22}]v_{2x} - \alpha H_y\sqrt{u}h_{2y} = 0$$

$$[(\gamma_{11} + i\gamma)u - \xi]v_{1y} + (\gamma_{12} - i\gamma)uv_{2y} + \alpha H_x\sqrt{u}h_{1y} = 0$$

$$(\gamma_{12} - i\gamma)uv_{1y} + (\gamma_{22} + i\gamma)uv_{2y} + \alpha H_x\sqrt{u}h_{2y} = 0, \qquad \eta_2 h_{1y} - \eta_1 h_{2y} = 0 \qquad (1.7)$$

$$(u + i\omega\eta)h_{1y} + \theta\eta_1\sqrt{u}[\eta_1(H_x v_{1y} - H_y v_{1x}) + \eta_2(H_x v_{2y} - H_y v_{2x})] = 0$$

$$[(\gamma_{11} + i\gamma)u - \xi]v_{1z} + (\gamma_{12} - i\gamma)uv_{2z} + \alpha H_x\sqrt{u}h_{1z} = 0$$

$$(\gamma_{12} - i\gamma)uv_{1z} + (\gamma_{22} + i\gamma)uv_{2z} + \alpha H_x\sqrt{u}h_{2z} = 0$$

$$(u + i\omega\eta)h_{1z} + \theta\eta_1 H_x\sqrt{u}(\eta_1 v_{1z} + \eta_2 v_{2z}) = 0, \qquad \eta_2 h_{1z} - \eta_1 h_{2z} = 0 \qquad (1.8)$$

where
$$\gamma_{11}=\frac{\rho_{11}}{\rho},\quad \gamma_{12}=\frac{\rho_{12}}{\rho},\quad \gamma_{22}=\frac{\rho_{22}}{\rho},\qquad \begin{aligned}\rho&=\rho_{11}+\rho_{22}+2\rho_{12}\\ \rho a^2&=\lambda+2\mu+R+2Q\end{aligned}$$
$$\sigma_{11}=\frac{\lambda+2\mu}{\rho a^2},\quad \sigma_{12}=\frac{Q}{\rho a^2},\quad \sigma_{22}=\frac{R}{\rho a^2},\quad \xi=\frac{\mu}{\rho a^2}$$
$$\alpha=\frac{1}{4\pi\rho a},\qquad \gamma=\frac{\beta}{\rho\omega},\qquad \eta=\alpha\vartheta\rho c^2(\eta_1+\eta_2),\qquad \vartheta=\frac{1}{a(\eta_1^2+\eta_2^2)}$$
The quantity $u=(\omega/ka)^2$ represents the square of the dimensionless phase velocity. The equations obtained can be arranged into two mutually independent groups, (1.7) and (1.8), and this implies that two independent classes of the magnetoacoustic waves exist. Waves belonging to the first class defined by Eqs. (1.7) are polarized in the $xy$-plane, while the waves of the second class described by (1.8) are polarized in the direction perpendicular to the $xy$-plane.

**2.** Let us consider the waves belonging to the first class. Equating the determinant of (1.7) to zero, we obtain the following dispersion equation:
$$(\eta_1^2+\eta_2^2)\Delta(u)\delta(u)(u+i\omega\eta)=\psi_x\Delta(u)(\Gamma u-\xi\eta_2^2)+\psi_y\ u\ \delta(u)(\Gamma u-\Sigma)\tag{2.1}$$
where
$$\Delta(u)=(\gamma_{11}\gamma_{22}-\gamma_{12}^2+i\gamma)u^2-(\gamma_{11}\sigma_{22}+\gamma_{22}\sigma_{11}-2\gamma_{12}\sigma_{12}+i\gamma)u+\sigma_{11}\sigma_{22}-\sigma_{12}^2$$
$$\delta(u)=(\gamma_{11}\gamma_{22}-\gamma_{12}^2+i\gamma)u-\xi(\gamma_{22}+i\gamma)$$
$$\psi_x=\alpha H_x^2/a,\qquad \Gamma=\eta_1^2(\gamma_{22}+i\gamma)+\eta_2^2(\gamma_{11}+i\gamma)-2\eta_1\eta_2(\gamma_{12}-i\gamma)$$
$$\psi_y=\alpha H_y^2/a,\qquad \Sigma=\eta_1^2\sigma_{22}+\eta_2^2\sigma_{11}-2\eta_1\eta_2\sigma_{12}$$

Eq. (2.1) is of the fourth degree in $u$, consequently four types of waves polarized in the $xy$-plane exist. Relations (1.7) yield
$$v_{1x}=\frac{a\psi_y\sqrt{u}}{\eta_1\Delta(u)}(\Gamma_2u-\Sigma_2)\frac{h_{1y}}{H_y},\qquad v_{2x}=\frac{\Gamma_1u-\Sigma_1}{\Gamma_2u-\Sigma_2}v_{1x},\qquad v_{1y}=-\frac{a\psi_x\sqrt{u}\Gamma_2}{\eta_1\delta(u)}\frac{h_{1y}}{H_x}\tag{2.2}$$
$$v_{2y}=\frac{\Gamma_1u-\xi\eta_2}{\Gamma_2u}v_{1y},\qquad \begin{aligned}\Gamma_1&=\eta_2(\gamma_{11}+i\gamma)-\eta_1(\gamma_{12}-i\gamma),\qquad &\Sigma_1&=\eta_2\sigma_{11}-\eta_1\sigma_{12}\\ \Gamma_2&=\eta_1(\gamma_{22}+i\gamma)-\eta_2(\gamma_{12}-i\gamma),\qquad &\Sigma_2&=\eta_1\sigma_{22}-\eta_2\sigma_{12}\end{aligned}$$
When the magnetic field is weak, $\psi=\psi_x+\psi_y\ll 1$ and the roots of (2.1) have the form
$$u^{(1)}=u_0^{(1)}+\frac{\psi_yu_0^{(1)}(\Gamma u_0^{(1)}-\Sigma)}{N(u_0^{(1)}-u_0^{(2)})(u_0^{(1)}-u_0^{(4)})},\quad u^{(2)}=u_0^{(2)}+\frac{\psi_yu_0^{(2)}(\Gamma u_0^{(2)}-\Sigma)}{N(u_0^{(2)}-u_0^{(1)})(u_0^{(2)}-u_0^{(4)})}$$
$$u^{(3)}=u_0^{(3)}+\frac{\psi_x(\Gamma u_0^{(3)}-\xi\eta_2^2)}{N(u_0^{(3)}-u_0^{(4)})},\quad N=(\eta_1^2+\eta_2^2)(\gamma_{11}\gamma_{22}-\gamma_{12}^2+i\gamma)$$
$$u^{(4)}=u_0^{(4)}+\frac{1}{N}\left[\frac{\psi_x(\Gamma u_0^{(4)}-\xi\eta_2^2)}{u_0^{(4)}-u_0^{(3)}}+\frac{\psi_yu_0^{(4)}(\Gamma u_0^{(4)}-\Sigma)}{(u_0^{(4)}-u_0^{(1)})(u_0^{(4)}-u_0^{(2)})}\right]\tag{2.3}$$
where $u_0^{(1)}$, $u_0^{(2)}$ and $u_0^{(3)}$ denote the squares of the dimensionless phase velocities of two longitudinal and one transverse wave occurring in the two-component medium in the absence of a magnetic field. Quantities $u_0^{(1)}$ and $u_0^{(2)}$ are the roots of the quadratic equation $\Delta(u)=0$ and $u_0^{(3)}$ and $u_0^{(4)}$ are given by
$$u_0^{(3)}=\frac{\xi(\gamma_{22}+i\gamma)}{\gamma_{11}\gamma_{22}-\gamma_{12}^2+i\gamma},\qquad u_0^{(3)}=-i\omega\eta$$
For the well conducting media we have $\omega\eta\ll 1$, and Formulas (2.3) somewhat simplify to yield

$$u^{(1)} = u_0^{(1)} + \frac{\psi_y (\Gamma u_0^{(1)} - \Sigma)}{N (u_0^{(1)} - u_0^{(2)})}, \qquad u^{(3)} = u_0^{(3)} + \frac{\psi_x (\Gamma u_0^{(3)} - \xi \eta_2^2)}{N u_0^{(3)}}$$
$$u^{(2)} = u_0^{(2)} + \frac{\psi_y (\Gamma u_0^{(2)} - \Sigma)}{N (u_0^{(2)} - u_0^{(1)})}, \qquad u^{(4)} = - i\omega\eta + \frac{\psi_x \eta_2^2}{(\eta_1^2 + \eta_2^2)(\gamma_{22} + i\gamma)} \tag{2.4}$$

The relations (2.2) yield, with the accuracy of up to the order of $\psi$,

$$v_{1x}^{(1)} = \frac{a (\eta_1^2 + \eta_2^2) \sqrt{u_0^{(1)}} (\Gamma_2 u_0^{(1)} - \Sigma_2)}{\eta_1 (\Gamma u_0^{(1)} - \Sigma)} \frac{h_{1y}^{(1)}}{H_y}, \qquad v_{1y}^{(1)} = 0$$
$$v_{1x}^{(2)} = \frac{a (\eta_1^2 + \eta_2^2) \sqrt{u_0^{(2)}} (\Gamma_1 u_0^{(2)} - \Sigma_1)}{\eta_1 (\Gamma u_0^{(2)} - \Sigma)} \frac{h_{1y}^{(2)}}{H_y}, \qquad v_{1y}^{(2)} = 0 \tag{2.5}$$
$$v_{1x}^{(3)} = 0, \qquad v_{1y}^{(3)} = - \frac{a (\eta_1^2 + \eta_2^2) (u_0^{(3)})^{3/2} \Gamma_2}{\eta_1 (\Gamma u_0^{(3)} - \xi \eta_2^2)} \frac{h_{1y}^{(3)}}{H_x}$$
$$v_{1x}^{(4)} = 0, \qquad v_{1y}^{(4)} = 0 \qquad v_{2x}^{(4)} = 0, \qquad v_{2y}^{(4)} = - \frac{a \psi_x}{(\gamma_{22} + i\gamma) \sqrt{u_0^{(4)}}} \frac{h_{2y}^{(4)}}{H_x}$$

In the approximation considered, first three waves represent, respectively, two longitudinal and one transverse wave, modified somewhat by the presence of the magnetic field. The fourth wave, in the case when the conductivity of the fluid component is much smaller than the conductivity of the elastic component, decays rapidly and is related to the process of diffusion of the magnetic field in the elastic component. In the opposite, limiting case when the conductivity of the fluid is infinite, this wave becomes the Alfven's wave for which the following equations hold:

$$u^{(4)} = \frac{\psi_x}{\gamma_{22} + i\gamma}, \qquad v_{2y}^{(4)} = - \frac{h_{2y}^{(4)}}{\sqrt{4\pi\rho (\gamma_{22} + i\gamma)}} \tag{2.6}$$

Neglecting the viscosity ($\gamma = 0$), we obtain the relations for the Alfven's waves [5].

**3.** Let us now consider the second class waves. Condition of compatibility of the system (1.8) leads to the following dispersion equation, second degree in $u$ :

$$(\eta_1^2 + \eta_2^2)\, \delta (u) (u + i\omega\eta) = \psi_x (\Gamma u - \xi \eta_2^2) \tag{3.1}$$

This shows that two types of waves exist, polarized in the direction perpendicular to the $xy$-plane, and from (1.8) we obtain the following relations for these two waves:

$$v_{1z} = - \frac{a \psi_x \sqrt{u} \Gamma_2}{\eta_1 \delta (u)} \frac{h_{1z}}{H_x}, \qquad v_{2z} = \frac{\Gamma_1 u - \xi \eta_2}{\Gamma_2 u} v_{1z} \tag{3.2}$$

When $\psi \ll 1$, then the roots of (3.1) have the form

$$u^{(5)} = u^{(3)}, \qquad u^{(6)} = u_0^{(4)} + \frac{\psi_x (\Gamma u_0^{(4)} - \xi \eta_2^2)}{N (u_0^{(4)} - u_0^{(3)})} \tag{3.3}$$

The root $u^{(5)}$ corresponds to the transverse wave, while $u^{(6)}$ to the Alfven's wave.
When the medium is a good conductor, $u^{(6)}$ coincides with $u^{(4)}$ and we obtain, with (2.5) taken into account, the following vector relation for the Alfven's waves

$$\mathbf{v}_2 = - \frac{a \psi_x}{(\gamma_{22} + i\gamma) \sqrt{u^{(4)}}} \frac{\mathbf{h}_2}{H_x} \tag{3.4}$$

**4.** In conclusion, we shall obtain the coefficients of absorption for the above waves. The ceofficient of absorption $\chi$ is given in the imaginary part of the wave number

$$\chi = \operatorname{Im} \frac{\omega}{a \sqrt{u}} \tag{4.1}$$

When $\gamma = \beta / \rho\omega \ll 1$, (2.4) yields with the accuracy of up to the principal terms,

$$\chi^{(1)} = \frac{\beta\,(u_0^{(1)} - 1)}{2\rho a\,(\gamma_{11}\gamma_{22} - \gamma_{12}^2)\,(u_0^{(1)} - u_0^{(2)})\sqrt{u_0^{(1)}}} \qquad (4.2)$$

$$\chi^{(2)} = \frac{\beta\,(u_0^{(2)} - 1)}{2\rho a\,(\gamma_{11}\gamma_{22} - \gamma_{12}^2)\,(u_0^{(2)} - u_0^{(1)})\sqrt{u_0^{(2)}}}, \qquad \chi^{(3)} = \frac{\beta\,(u_0^{(3)} - \xi)}{2\rho a\xi\gamma_{22}\sqrt{u_0^{(3)}}}$$

$$\chi^{(4)} = \frac{\omega}{a}\left\{\frac{\gamma_{22}(\eta_1^2 + \eta_2^2)}{2}\left[\frac{\sqrt{\psi_x^2\eta_2^4 + (\omega\eta\gamma_{22})^2(\eta_1^2 + \eta_2^2)^2} - \psi_x\eta_2^2}{\psi_x^2\eta_2^4 + (\omega\eta\gamma_{22})^2(\eta_1^2 + \eta_2^2)^2}\right]\right\}^{1/2}$$

where the values of $u_0^{(\nu)}$ are taken at $\gamma = 0$. In the other limiting case when $\gamma \gg 1$ we have

$$\chi^{(1)} = \frac{\rho\omega^2}{2a\beta}(\gamma_{11}\sigma_{22} + \gamma_{22}\sigma_{11} - 2\gamma_{12}\sigma_{12} - \gamma_{11}\gamma_{22} + \gamma_{12}^2 - \sigma_{11}\sigma_{22} + \sigma_{12}^2)$$

$$\chi^{(2)} = \frac{1}{a}\left(\frac{\beta\omega}{2\rho\,(\sigma_{11}\sigma_{22} - \sigma_{12}^2)}\right)^{1/2}, \qquad \chi^{(3)} = \frac{\rho\omega^2}{2a\beta\sqrt{\xi}}(\gamma_{22} - \gamma_{11}\gamma_{22} + \gamma_{12}^2) \qquad (4.3)$$

$$\chi^{(4)} = \frac{1}{a}\left(\frac{\beta\,(\eta_1^2 + \eta_2^2)\,\omega}{2\,[\beta\eta\,(\eta_1^2 + \eta_2^2) + \rho\eta_2^2\psi_x}\right)^{1/2}$$

Thus we see that in the critical cases shown above, magnetic field has no effect on the coefficients of absorption of the first three waves. Their decay is caused mainly by the viscous dissipation. Absorption of the fourth wave depends on the magnetic field, conductivities of the medium components and of the viscosity of the fluid. Coefficients of absorption of the waves polarized perpendicularly to the $xy$-plane, coincide, respectively, with $\chi^{(3)}$ and $\chi^{(4)}$.

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## BIBLIOGRAPHY

1. Frenkel', Ia. I., On the theory of the seismic and electroseismic phenomena in a moist soil. Izv. Akad. Nauk SSSR, seriia geogr. i geofiz., Vol. 8, №4, 1944.
2. Biot, M. A., Theory of elasticity and consolidation for a porous anisotropic solid. J. Appl. Phys., Vol. 26, №2, 1955.
3. Biot, M. A., Theory of propagation of elastic waves in a fluid-saturated porous solid. 1. Low-frequency range. J. Acoust. Soc. Am., Vol. 28, №2, 1956.
4. Kosachevskii, L. Ia., On the propagation of elastic waves in two-component media. PMM Vol. 23, №6, 1959.
5. Landau, L. D. and Lifshits, E. M., Electrodynamics of Continuous Media. M., Gostekhizdat, 1957.

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